by means of invariants. The case of exception is the biquadratic equation, for which it is impossible to assign an invariantive criterion that shall serve to distinguish between the case of all the roots being real and all imaginary.

It is proper to notice that it follows, from the definition of the symbol $m.\mu$, that its value is zero whenever m is less than 2μ. Thus, in the matrix written out above, the symbols 3·2, 4·3, 5·3, 5·4, 6·4, 7·4 may be replaced by zeros.

The above general result for a curve of any order is actually obtained by a far less expenditure of thought and labour than was employed by Monge, Halphen, and others to obtain it for the trifling case of a conic. I touch a secret spring, and the doors of the cabinet fly wide open.[1] J. J. SYLVESTER

New College, Oxford, August 6

CAPILLARY ATTRACTION[2]

III.

IN these other diagrams, however (Figs. 13 to 28), we have certain portions of the curves taken to represent real capillary surfaces shown in section. In Fig. 13 a solid sphere is shown in four different positions in contact with a mercury surface; and again, in Fig. 14 we have a section of the form assumed by mercury resting in a circular V-groove. Figs. 15 to 28 show water-surfaces under different conditions as to capillarity; the scale of the drawings for each set of figures is shown by a line the length of

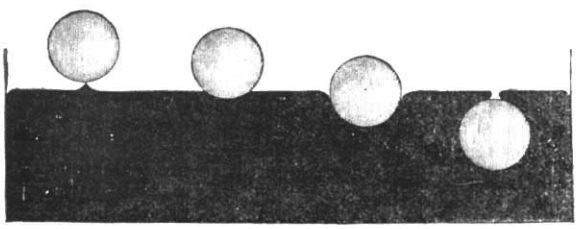


FIG. 13.—Mercury in contact with solid spheres (say of glass).

which represents one centimetre; the dotted horizontal lines indicate the positions of the free water-level. The drawings are sufficiently explicit to require no further reference here save the remark that *water* is represented by the lighter shading, and *solid* by the darker.

We have been thinking of our pieces of rigidified water as becoming suddenly liquified, and conceiving them inclosed within ideal contractile films; I have here an arrangement by which I can exhibit on an enlarged

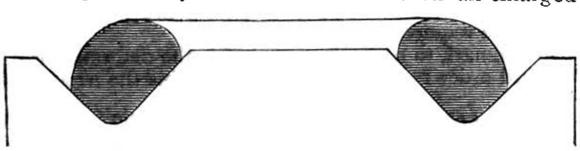
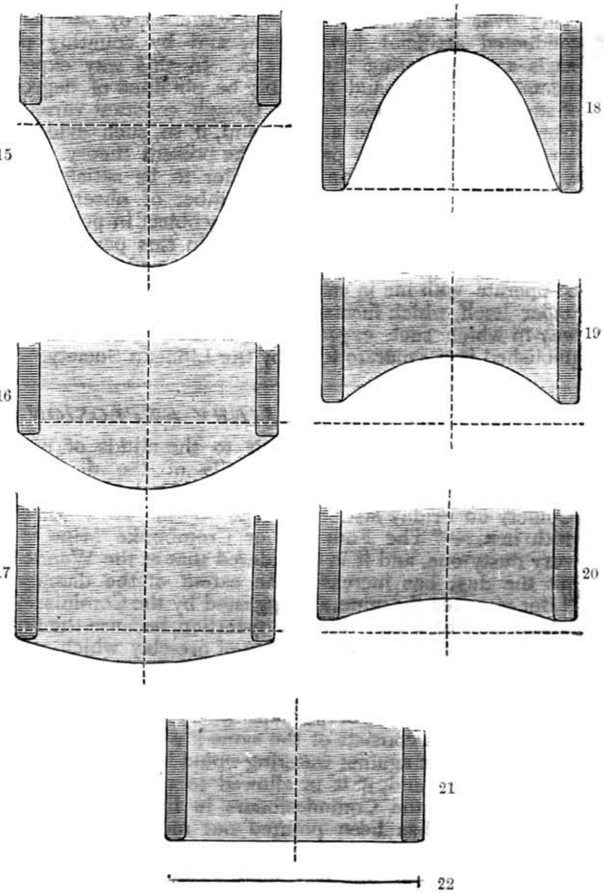


FIG. 14.—Sectional view of circular V-groove containing mercury.

scale a pendant drop, inclosed not in an *ideal* film, but in a *real* film of thin sheet india-rubber. The apparatus which you see here suspended from the roof is a stout metal ring of 60 centimetres diameter, with its aperture closed by a sheet of india-rubber tied to it all round, stretched uniformly in all directions, and as tightly as could be done without special apparatus for stretching it and binding it to the ring when stretched.

I now pour in water, and we find the flexible bottom assuming very much the same shape as the drop which you saw hanging from my finger after it had been dipped into and removed from the vessel of water (see Fig. 16).



FIGS. 15-21.—Water in glass tubes, the internal diameter of which may be found from Fig. 22, which represents a length of one centimetre.

I continue to pour in more water, and the form changes gradually and slowly, preserving meanwhile the general form of a drop such as is shown in Fig. 15, until, when a certain quantity of water has been poured in, a sudden change takes place. The sudden change corresponds to the breaking away of a real drop of water from, for example, the mouth of a tea-urn, when the stopcock is so nearly closed that a very slow dropping takes place. The drop in the india-rubber bag, however, does not fall away, because the tension of the india-rubber increases enormously when

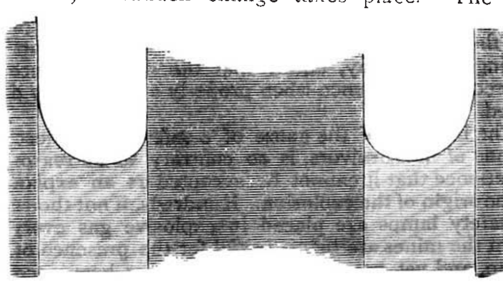


FIG. 23.—Water resting in the space between a solid cylinder and a concentric hollow cylinder.

[1] Adopting the convention for degree and weight of a differential coefficient usual in the theory of reciprocants the deg : weight of the differential criterion of the nth order will be easily found to be—

$$\frac{n.\overline{n+1}.\overline{n+2}}{6} : \frac{\overline{n-1}.n.\overline{n+1}.\overline{n+2}}{8}$$

except that for $n=2$ it is 3 : 3 instead of 4 : 3.

[2] Lecture delivered at the Royal Institution. Revised and extended by the Author. Continued from p. 294.

the india-rubber is stretched. The tension of the real film at the surface of a drop of water remains constant, however much the surface is stretched, and therefore the drop breaks away instantly when enough of water has been supplied from above to feed the drop to the greatest volume that can hang from the particular size of tube which is used.

I now put this siphon into action, gradually drawing off some of the water, and we find the drop gradually diminishes until a sudden change again occurs and it assumes the form we observed (Fig. 16) when I first poured in the water. I instantly stop the action of the siphon, and we now find that the great drop has two possible forms of stable equilibrium, with an unstable form intermediate between them.

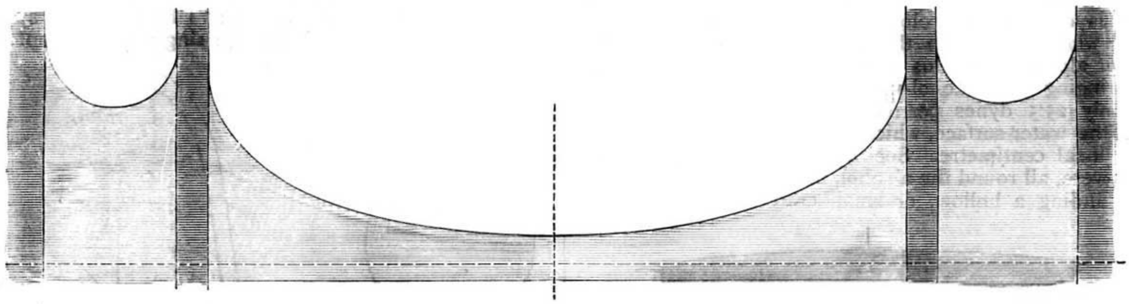


FIG. 24 —Water resting in two co-axial cylinders; scale is represented by Fig. 28.

Here is an experimental proof of this statement. With the drop in its higher stable form I cause it to vibrate so as alternately to decrease and increase the axial length, and you see that when the vibrations are such as to cause the increase of length to reach a certain limit there is a sudden change to the lower stable form, and we may

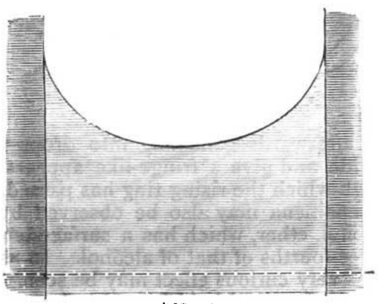


FIG. 25.

now leave the mass performing small vibrations about that lower form. I now increase these small vibrations, and we see that, whenever, in one of the upward (increasing) vibrations, the contraction of axial length reaches the limit already referred to, there is again a sudden change, which I promote by gently lifting with my hands, and the mass assumes the higher stable form, and we have it again performing small vibrations about this form.

The two positions of stable equilibrium, and the one of unstable intermediate between them, is a curious peculiarity of the hydrostatic problem presented by the water supported by india-rubber in the manner of the experiment.

I have here a simple arrangement of apparatus (Figs. 29 and 30) by which, with proper optical aids, such as a cathetometer and a microscope, we can make the necessary measurements on real drops of water or other liquid, for the purpose of determining the values of the capillary constants. For stability the drop hanging from the open tube should be just less than a hemisphere, but for convenience it is shown, as in the enlarged drawing of the nozzle (Fig. 30), exactly hemispherical. By means of the siphon the difference of levels, h, between the free level surface of the water in the vessel to which the nozzle is attached, and the lowest point in the drop hanging from the nozzle, may be varied, and corresponding measurements taken of h and of r, the radius of curvature of the drop at its lowest point. This measurement of the curvature of the drop is easily made with somewhat close accuracy, by known microscopic methods. The surface-tension T of the liquid is calculated from the radius, r, and the observed difference of levels, h, as follows:—

$$\frac{2T}{r} = h;$$

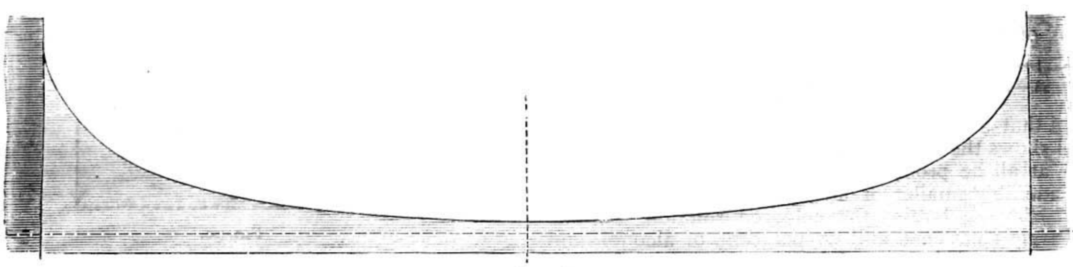


FIG. 26.

FIGS. 25 and 26.—Water resting in hollow cylinders (tubes); scale is represented by Fig. 28.

for example, if the liquid taken be water, with a free-surface tension of 75 milligrammes per centimetre, and $r = {\cdot}05$ cm., h is equal to 3 centimetres.

Many experiments may be devised to illustrate the effect of surface-tension when two liquids, of which the surface-tensions are widely different, are brought into contact with each other. Thus we may place on the surface of a thin layer of water, wetting uniformly the surface of a glass plate or tray, a drop of alcohol or ether, and so cause the surface-tension of the liquid layer to become smaller in the region covered by the alcohol or ether. On the other hand, from a surface-layer of alco-

hol largely diluted with water we may arrange to withdraw part of the alcohol at one particular place by promoting its rapid evaporation, and thereby increase the surface-tension of the liquid layer in that region by diminishing the percentage of alcohol which it contains.

In this shallow tray, the bottom of which is of ground glass resting on white paper, so as to make the phenomena to be exhibited more easily visible, there is a thin layer of water coloured deep blue with aniline; now, when I place on the water-surface a small quantity of alcohol from this fine pipette, observe the effect of bringing the alcohol-surface, with a surface-tension of only 25·5 dynes per lineal centimetre, into contact with the water-surface, which has a tension of 75 dynes per lineal centimetre. See how the water pulls back, as it were, all round the alcohol, forming a circular ridge surrounding a hollow, or small crater, which gradually

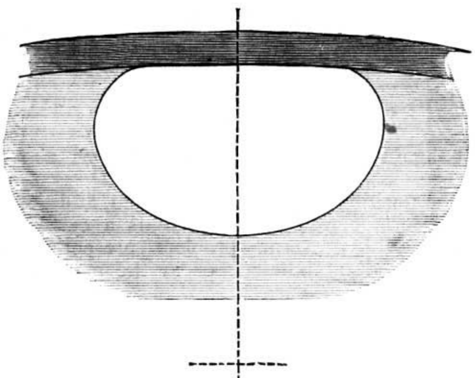


FIG. 27.—Section of the air-bubble in a level tube filled with water, and bent so that its axis is part of a circle of large radius; scale is represented in Fig. 28.

widens and deepens until the glass plate is actually laid bare in the centre, and the liquid is heaped up in a circular ridge around it. Similarly, when I paint with a brush a streak of alcohol across the tray, we find the water drawing back on each side from the portion of the tray touched with the brush. Now, when I incline the glass tray, it is most interesting to observe how the coloured water with its slight admixture of alcohol flows down the incline—first in isolated drops, afterwards joining together into narrow continuous streams.

These and other well-known phenomena, including that interesting one, "tears of strong wine," were described and explained in a paper "On Certain Curious Motions Observable on the Surfaces of Wine and other Alcoholic Liquors," by my brother, Prof. James Thomson, read before Section A of the British Association at the Glagow meeting of 1855.

FIG. 28.—Represents a length of one centimetre for Figs. 24 to 27.

I find that a solution containing about 25 per cent. of alcohol shows the "tears" readily and well, but that they cannot at all be produced if the percentage of alcohol is considerably smaller or considerably greater than 25. In two of those bottles the coloured solution contains respectively 1 per cent. and 90 per cent. of alcohol, and in them you see it is impossible to produce the "tears"; but when I take this third bottle, in which the coloured liquid contains 25 per cent. of alcohol, and operate upon it, you see—there—the "tears" begin to form at once. I first incline and rotate the bottle so as to wet its inner surface with the liquid, and then, leaving it quite still, I remove the stopper, and withdraw by means of this paper tube the mixture of air and alcoholic vapour from the bottle and allow fresh air to take its place. In this way I promote the evaporation of alcohol from all liquid surfaces within the bottle, and where the liquid is in the form of a thin film it very speedily loses a great part of its alcohol. Hence the surface-tension of the thin film of liquid on the interior wall of the bottle comes to have a greater and greater value than the surface-tension of the mass of liquid in the bottom, and where these two liquid surfaces, having different surface-tensions, come together we have the phenomena of "tears." There, as I hasten the evaporation, you see the horizontal ring rising up the side of the

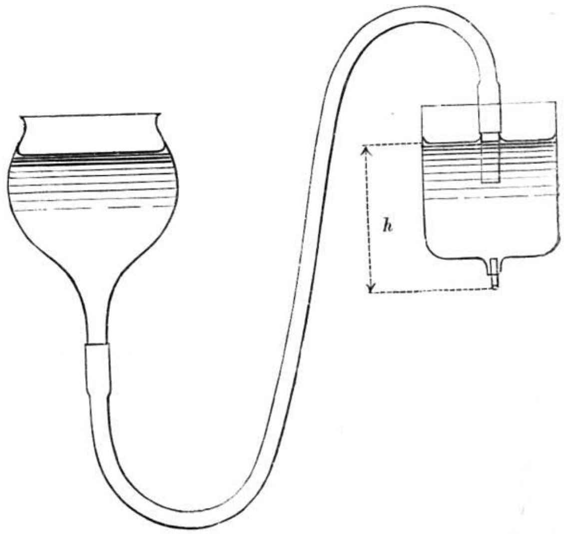


FIG. 29.

bottle, and afterwards collecting into drops which slip down the side and give a fringe-like appearance to the space through which the rising ring has passed.

These phenomena may also be observed by using, instead of alcohol, ether, which has a surface-tension equal to about three-fourths of that of alcohol. In using ether, however, this very curious effect may be seen.[1] I dip the brush into the ether, and hold it near to but not touching the water-surface. Now I see a hollow formed, which becomes more or less deep according as the brush is

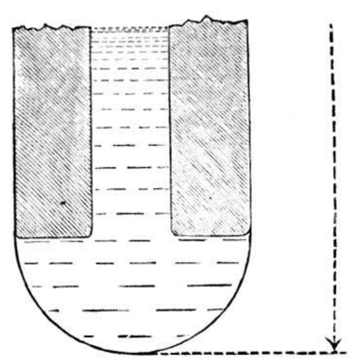


FIG. 30.

nearer to or farther from the normal water surface, and it follows the brush about as I move it so.

Here is an experiment showing the effect of heat on surface tension. Over a portion of this tin plate there is a thin layer of resin. I lay the tin plate on this hot copper cylinder, and we at once see the fluid resin drawing back from the portion of the tin plate directly over the end of the heated copper cylinder, and leaving a

[1] See Clerk-Maxwell's article (p. 65) on "Capillary Attraction" ("Encyclopædia Britannica," 9th edition).

circular space on the surface of the tin plate almost clear of resin, showing how very much the surface-tension of hot resin is less than that of cold resin.

Note of January 30, 1886.—The equations (8) and (9) on p. 59 of Clerk-Maxwell's article on "Capillary Attraction" in the ninth edition of the "Encyclopædia Brittanica" do not contain terms depending on the mutual action between the two liquids, and the concluding expression (10), and the last small print paragraph of the page are wholly vitiated by this omission. The paragraph immediately following equation (10) is as follows:—

"If this quantity is positive, the surface of contact will tend to contract, and the liquids will remain distinct. If, however, it were negative, the displacement of the liquids which tends to enlarge the surface of contact would be aided by the molecular forces, so that the liquids, if not kept separate by gravity, would become thoroughly mixed. No instance, however, of a phenomenon of this kind has been discovered, for those liquids which mix of themselves do so by the process of diffusion, which is a molecular motion, and not by the spontaneous puckering and replication of the boundary surface as would be the case if T were negative."

It seems to me that this view is not correct; but that on the contrary there is this "puckering" as the *very beginning* of diffusion. What I have given in the lecture as reported in the text above seems to me the right view of the case as regards diffusion in relation to interfacial tension.

It may also be remarked that Clerk-Maxwell, in the large print paragraph of p. 59, preceding equation (1), and in his application of the term potential energy to E in the small print, designated by *energy* what is in reality exhaustion of energy or negative energy; and the same inadvertence renders the small print paragraph on p. 60 very obscure. The curious and interesting statement at the top of the second column of p. 63, regarding a drop of carbon disulphide in contact with a drop of water in a capillary tube would constitute a perpetual motion if it were true for a tube not first wetted with water through part of its bore—". . . if a drop of water and a drop of bisulphide of carbon be placed in contact in a horizontal capillary tube, the bisulphide of carbon will chase the water along the tube."

Additional Note of June 5, 1886.—I have carefully tried the experiment referred to in the preceding sentence, and have not found the alleged motion.

WILLIAM THOMSON

OUR FOSSIL PSEUDO-ALGÆ

DURING the last half-century many palæontologists have described anomalous objects, some of which have been regarded as fossil marine Algæ, and others as tracks of various marine invertebrate animals; and since the publication of Darwin's theory of evolution various attempts have been made to utilise some of these in formulating a pedigree for the living types of vegetation. Amongst those who have tried to accomplish this object my distinguished friend the Marquis of Saporta, and his colleague, M. Marion, occupy the most prominent position. They have in several publications described and figured many objects which they believe to have been true marine Algæ, and out of which they have constructed the lower roots of their genealogical tree. But meanwhile there has grown up an enlarging school of palæontologists who look with strong suspicion upon these genealogies; who refuse to recognise the vegetable character of these objects; who believe most of them to be casts of various ridges and furrows, most of which have been tracks produced by creeping invertebrate animals or by the still more mechanical agencies of wind and water. At the head of this school Prof. Nathorst, of Stockholm, stands pre-eminent. An animated controversy sprang up some time ago between M. Nathorst and M. Saporta relative to this subject. Blast and counterblast have succeeded one another, and the latest discharge of palæo-botanical artillery has just been fired off by M. Nathorst in the form of a memoir entitled "Nouvelles Observations sur des Traces d'Animaux et autres Phénomènes d'Origine purement méchanique décrits comme 'Algues Fossiles.'"

Enjoying the privilege of an intimate and valued friendship with both these distinguished palæontologists, I am anxious to do full justice to both. But I must admit that my judgment and experiences bring me into closer agreement with the northern naturalist than with his French antagonist. The interesting subject discussed by them has long occupied my attention, and my conclusions respecting it have not been hastily arrived at.

The question in debate is not whether or not marine Algæ existed in Palæozoic and later geological epochs: on this point Nathorst and Saporta are agreed. The abundance of phytophagous marine mollusks found even in the Cambrian, as in most of the other fossiliferous strata, clearly demonstrates that there must have been submarine pastures upon which they could feed. The question is, are numerous objects, found in strata of marine origin, and believed by some to be fossilised marine Algæ, really such? To this query Saporta answers *Yes;* Nathorst's reply is an emphatic *No.* Hence the elaborate controversial literature of which these two *savants* are the authors. To condense their several articles into an abstract is not easy, but such an abstract of M. Nathorst's latest publication may be attempted, illustrating the general features of the discussion.

Throughout his memoir M. Nathorst rests prominently upon two general propositions which appear to me to be unanswerable. The first is that all or nearly all these debatable pseudo-Algæ stand out in bold demi-relief from the *inferior* surfaces of the rocky slabs to which they are attached, and that beyond their sculptured surfaces they as constantly consist of a mere extension of the rock of which they form a part. Hence Nathorst insists that they are merely convex casts of what were primarily concave grooves or channels on the surface of the subjacent stratum.

In reply to this opinion M. Saporta publishes figures of casts of vegetable fragments in demi-relief, the positions of which on the inferior surfaces of slabs are identical with those of the pseudo-Algæ under discussion. One of these is a fragment of what appears to be a twig of a Conifer, of which the lower side alone is preserved in demi-relief. Nathorst freely admits the possible existence of such specimens, but he regrets Saporta's explanation of them. *Imprimis,* he affirms with inexorable logic, that such examples are so rare and exceptional that they only prove the opposite of the rule which they are alleged by Saporta to sustain. Whenever fragments like these are found embedded in the rocks, they almost invariably display traces of both their upper and lower surfaces; whereas this is scarcely ever the case with the disputed Fucoids, and in the very few instances where such are supposed to have been met with, their entirely exceptional character suggests a very different explanation of them from that proposed by M. Saporta.

It is difficult to understand how a cylindrical object sufficiently dense to produce a deep concave impression upon hardening mud could do so without leaving some trace of its upper surface upon the opposite surface of the sand by which that mud became overlain. Saporta's theory explaining why it does not do so is surely untenable. That theory supposes that an organism half embedded in mud and overlain by sand began to decay at its *upper* surface, which decay ultimately reached the lower surface which rested on the mud; that, as the decay proceeded downwards, the superimposed sand would finally reach the concave mould in the mud which it would fill, and